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The Tangent Rule in Practice

Experimental Work to Test the Theory

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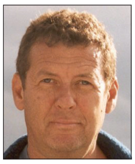

The tangent rule history is centuries old on the theory side: 200 years ago Poisson investigated the pendulum with small parameter expansions just like the ones we use today to make high Q systems manageable for digital computation[1]. To learn about it in an informal, nevertheless rigorous, way, Philip Woodward wrote a highly readable chapter on this subject in *My Own Right Time*[2].

The theory is so general and simple that probably little attention has been given to measuring its predictions explicitly. However, being new to this subject, I might have missed other attempts. Taking advantage of the simplicity and inexpensiveness of the necessary equipment, this work shows the period dependence from the phase of the escapement impulse in a particularly simple and prototypical case from data collected in a single experimental run.

The tangent rule says that any time a force is applied to a pendulum its oscillation period is affected, unless the precaution is taken of keeping the impulse symmetrical with respect to the pendulum rest position. This applies to all forces: the ones that keep the clock in motion, as well as the equally powerful ones which slow it down.

For this experimental work a Synchronome clock has been modified, inhibiting its traditional mechanical escapement and adding a coil which impulses the iron bob of the clock, **Figure 1**. The same image shows the arrangement of the optical-sensor just below the bob at the pendulum vertical.

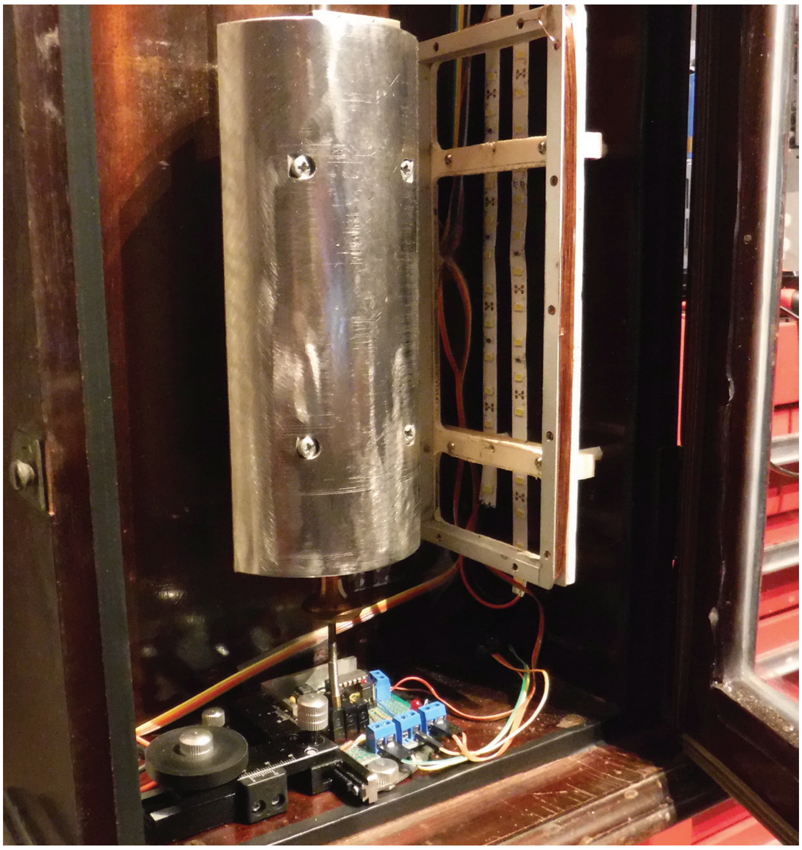

Figure 1. All period measurements for this work have been performed with a timer equipped with a 10 MHz thermally controlled quartz oscillator. The timer, a kit designed by Luke Mester[3], unloads its measurements to an IBM pc for storage and computation.

For these measurements the pendulum is powered every five periods, as opposed to the 15 periods of the original clock. The current pulse in the coil is triggered by the same photo-interrupt switch used for diagnostic purposes via a quartz Arduino micro-controller.

Impulsing the clock every few periods is particularly useful in this case since it allows us to compare the impulsed period with the free oscillations frequently, to improve statistics – and with the two events in close proximity – before other disturbances can interfere.

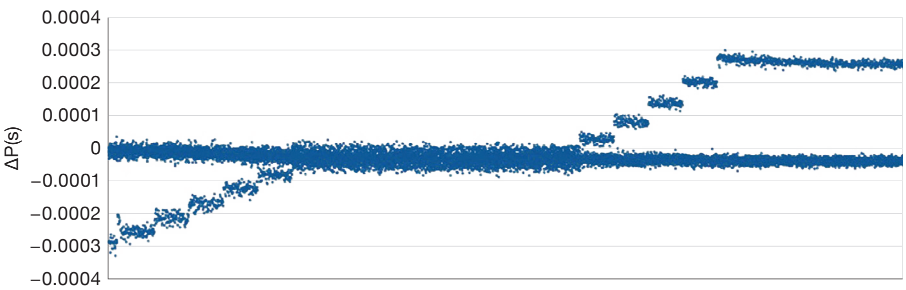

Figure 2. The roughly five-hour collection of single period fluctuations from average (= 2 s), against experimental time, every dot is one period.

For this work the power pulse duration has been kept constant at 65 ms and the centre pulse delay has been programmed with respect to the right to left pendulum centre crossing with a delay ranging from 900 to 1100 ms in steps of 20 ms. **Figure 2** shows the roughly five-hour collection of single period fluctuations from average (= 2 s), against experimental time, every dot one period.

The step-like regions in the plot correspond to those periods that receive an impulse with one of the ten different delays mentioned above. The typical period standard deviation for the whole run was 10 μs.

The maximum temperature swing during this five-hour long, automatically run, experiment was 0.2 °C and the full pendulum swing approximately four degrees. The average free pendulum Q, measured at the end of this experimental run, was 5500 at 3.6 to 4.4 degrees of amplitude.

Figure 3 shows the detail of typical period sequences belonging to the same dataset as **Figure 2** for different impulse delays with respect to centre. In summary, **Figure 4** shows the impulse period deviation in seconds versus the impulse delay.

The remarkable agreement of these experimental results with the theory prediction from Philip Woodward is shown in **Figure 5** (n is the number of periods in a clock impulsing cycle, n = 5 in this case).

REFERENCES

1. Bogoliubov, N. N., Mitropolsky, Y. A., *Asymptotic Methods in the Theory of Non-Linear Oscillations* (State Publishing House for Technical-Theoretical Literature, 1961).
2. Woodward, P., *My Own Right Time: An Exploration of Clockwork Design* (OUP Oxford, 1995).
3. Luke Mester <https://mesterhome.com/timer/index.html> (accessed 1 June, 2023).

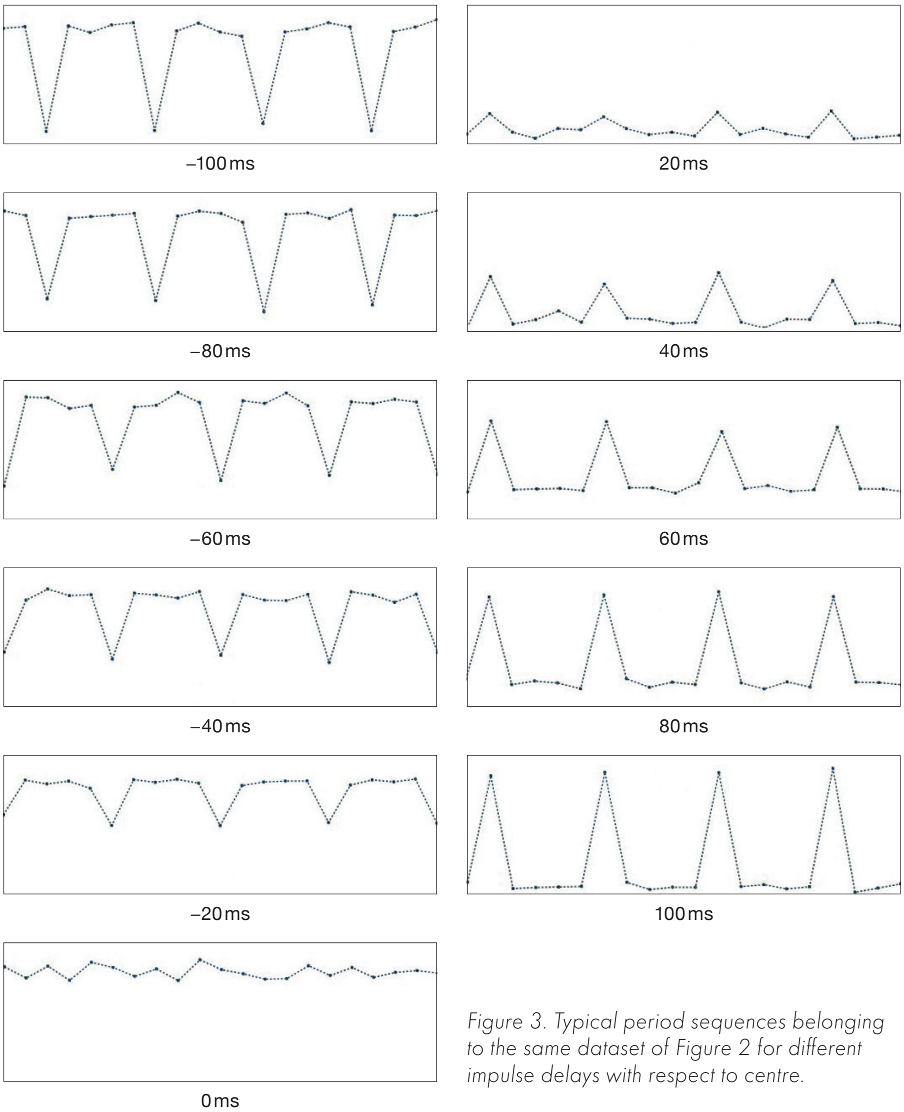

Figure 3. Typical period sequences belonging to the same dataset of Figure 2 for different impulse delays with respect to centre.

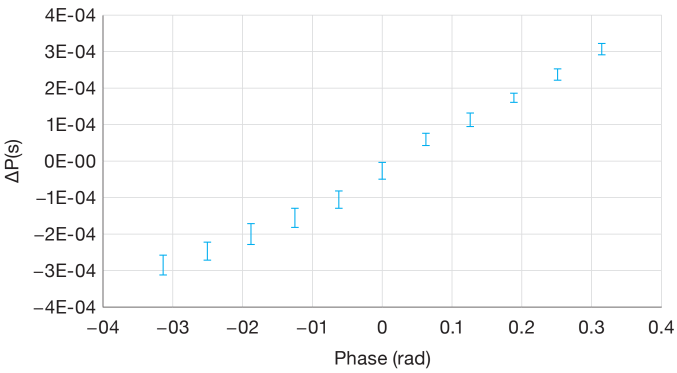

Figure 4. The impulse period deviation in seconds versus the impulse delay.

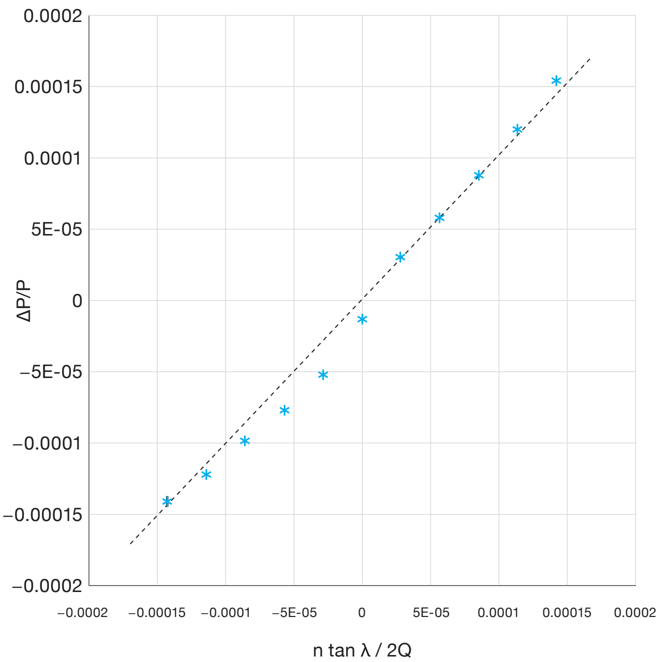

Figure 5. The agreement of the experimental results with the theory prediction from Philip Woodward.