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A Pendulum with Parametric Excitation

An Unusual New System

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Introduction

The September 1987 issue of *The Horological Journal* featured an article by Peter B. Wills MBHI, entitled 'The Free Dynadromic Pendulum'[1]. To the best of our knowledge, Peter's clock is unique: it is not driven by an escapement or a solenoid. Instead, it is energised by periodically lifting and dropping the entirety of the pendulum, including the support.

Peter's pendulum falls into a class of 'parametric oscillators'. These derive their energy from the variation of one of their basic system parameters[2], as opposed to being driven by an external action.

From the equation for the beat time $T = \pi\sqrt{(l/g)}$ (using T here as a single beat rather than the usual period which would be double) we know that the basic parameters of a pendulum are l and g, respectively the length and the acceleration due to gravity.In Peter's pendulum, the carefully timed accelerations of the lifting and dropping have the effect of seemingly varying g.

In this article, I describe a parametrically driven pendulum of an alternative type. It is energised by variation of its length l, namely by raising and lowering the bob in synchronicity with the beat. **Figure 1** shows the overall construction of a one-second pendulum. The set-up is experimental and prototypical, largely made of materials and components lying around in the workshop.

Figure 1. Overall view of the pendulum – its experimental nature is evident.

Parametric excitation is extensively studied in the scientific domain, often in relation to the playground swing problem, or to coupled pendulums and chaos theory[3,4]. Parametric oscillation, or parametric amplification, is used in a variety of devices. One can excite LC oscillators by variation of the capacitance C, using a varactor (a varactor is a type of diode, used as a variable capacitor by varying the voltage across it). Optical and microwave parametric oscillators and amplifiers are well known. However, to my knowledge, no serious attempt has been made to use variation of length for driving a horological pendulum. Luckily, the application to a horological pendulum requires only the very simplest part of the theory.

How This Pendulum Works

To gain an understanding of how this works, we can think of the pendulum as a child on a swing[3]. In this analogy, normal pendulums correspond to children being pushed by parents, who play the role of the escapement or solenoid. In contrast, parametrically driven pendulums correspond to children who swing themselves.

The easiest analogy is with swinging in standing position. The child stands up (raises the centre of gravity) when the swing is at the bottom point, and squats down (lowers the centre of gravity) at the extremes of the swing. In doing so, it performs mechanical work, which energises the swing.

Similarly, as shown in **Figure 2**, our pendulum contains a mechanism – in this case a stepper-motor linear actuator – which raises and lowers a moving part of the bob mass in synchronicity with the swing. It raises the bob near the

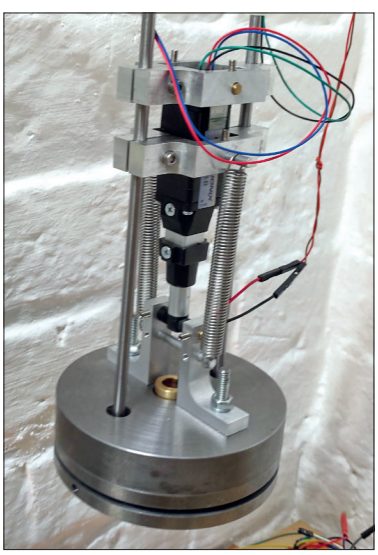

Figure 2. Detail of the raising/lowering mechanism.

bottom point of the swing, and lowers it near the amplitude point. Like the child, the mechanism performs mechanical work, which energises the pendulum.

Let's analyse an idealised model of a simple pendulum in which the bob-raisings and lowerings occur instantaneously. Then, at the bottom point, the mechanism performs mechanical work against the sum of the centrifugal force and gravity. Whereas, at the extremes of swing, there is no centrifugal force, so the mechanism recovers less mechanical work than it has performed at the bottom point.

The result is a positive net amount of work. For the pendulum motion to be sustained, or grow, this work must offset the mechanical losses, as expressed through the quality factor Q. The 'Appendix' at the end of this feature gives the calculation in some detail. The result is that for this idealised pendulum, with a quality factor Q, to swing sustainably, the minimum length variation Δl is given by:

$$\frac{\Delta l}{l} \geq \frac{\pi}{3Q} \qquad (1)$$

This gives a value of about 0.2 mm for a 1 s (or 1 m) pendulum with a Q of 5000. In reality, any moving body has limits on velocity and acceleration, so that the raising and lowering are spread over an arc of the beat. Larger values for Δl, in our case around 0.5 to 1 mm, are then required.

Two more important features are to be noted. Firstly, the excitation frequency of a parametric device is always double the system frequency: the child stands/squats once per beat, hence twice for one full swing period.

Secondly, because the energy gain ΔE is linearly proportional to the energy E itself, E will grow exponentially for a given Δl. If continually excited, the amplitude will keep increasing, until some non-linearity sets in, or something breaks. It is, therefore, necessary to apply feedback control over the amplitude. Once the pendulum reaches a certain required amplitude, the feedback system stops the excitation. The amplitude then decays, and the feedback system resumes excitation. With the current programming, we use sequences of 10 excitation beats with Δl of 1 mm. These will then be followed by around 13 idle beats.

Finally, we must examine the equivalent of the escapement effect, i.e. the effect of the excitation on the beat-time. Firstly, we now have a pendulum with two lengths. Secondly, when excitation is active, we are adding kinetic energy throughout the lower part of the beat; an action that is directly equivalent to the action of an escapement or solenoid.

To minimise these effects, we adopt a more sophisticated excitation strategy than that described for the child on the swing. Instead of simply moving the bob 'up, down', we adopt a strategy of 'half-down 1 (at one extreme of swing); full-up (around the bottom point); half-down 2 (at the opposite extreme of swing)'.

This largely cancels the two-lengths effect because the average length for an excited beat and an idle beat are nearly the same. As a further step, reducing 'half-down 1' and increasing by an equal amount 'half-down 2' will slightly reduce the time of an excited beat, and vice-versa. With the help of accurate measurements, we can equalise the durations of excited and idle beats, thereby eliminating any 'escapement effect'.

Technical Details

In this section we provide further technical details. **Figure 2** shows the essentials of the mechanism. As mentioned previously, it consists of a stepper-motor linear actuator, supported on the 5 mm pendulum rods. Its moving end is connected to the 'moving mass', the thicker of the two steel cylindrical discs, and it can move this up and down. Two springs are added in order to reduce the load on the actuator. At the bottom we see the fixed mass, the shorter of the two discs. The central bronze tube serves as a slide bearing for the moving mass and as a mount for a 1 mW solid-state laser, which points downwards.

Figure 3 illustrates the top of the pendulum. It shows a standard support, a spring made of feeler gauges, and the thin wires that power the actuator and the laser-diode.

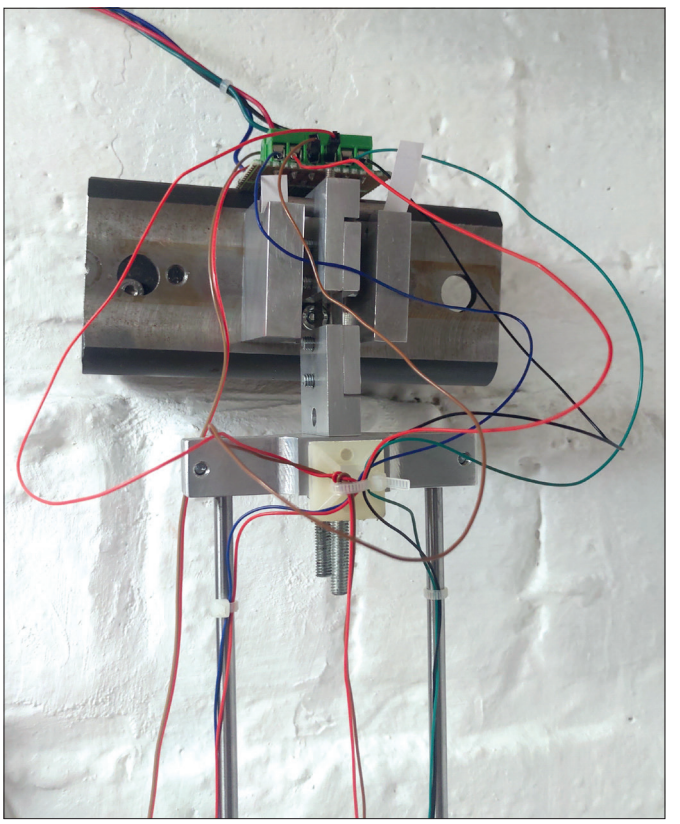

Figure 3. Detail of the support, pendulum spring and wires that power the stepper motor and laser-diode.

The laser-diode points down on to an array of five photo-diodes, as shown in **Figure 4**. These correspond to bottom, left and right extremes of swing, and two intermediate swing angles. The circuit board contains simple signal-processing electronics. The figure also shows the first of two system-on-chip boards (an Arduino *Due*).

Figure 5 shows a block diagram of the control system. The photo-diode signals feed into the first system-on-chip board. Based on the detector signals, and using noise and interference limiting measures, this system initiates the up/down movements of the 'movable mass'. It decides when excitations are necessary, and so implements the amplitude feedback. It passes requests for 'half-down1', full up', 'half-down 2' to a second system-on-chip board (an Arduino *Uno*), which is responsible for generating the motion sequences for the stepper motor.

To obtain smooth motor movements, these sequences need to involve acceleration and deceleration. This second system-on-chip board passes step/direction signals to the stepper motor's power driver which, in turn, energises the two stepper motor windings.

Both Arduinos interface with a PC, via USB serial connections. The PC runs a dedicated application that allows manual control of the stepper motor position, needed to set the pendulum's mid-position. It also displays extensive information for each beat, and writes log files which are subsequently analysed using MATLAB.

Initial Results

The pendulum has been operating now for a couple of weeks in the current configuration, and with the current software version.

First results were obtained from calibration against Global Positioning System (GPS) time, using a small GPS module with 'pulse-per-second' output. **Figure 6** shows that, over a period of one-week, the deviation between the pendulum and the GPS, sampled hourly, varies between +370 ms and −770 ms.

The variation is smooth, and is no doubt mostly driven by temperature changes. Given the experimental nature of the setup, and the absence of any temperature compensation, we may conclude these results are sufficiently encouraging to continue exploration, and to develop an improved version of the pendulum.

Secondly, we can clearly show the effect of the excitation strategy – the 'half-downs' versus the 'ups' – on the 'escapement effect'. **Figure 7** shows the duration of 15,000 periods (30,000 beats) for an excitation where 'half-down 1' is 0.525 mm, 'half-down 2' is 0.475 mm and 'up' is, of course, 1mm. The green dots represent idle periods, i.e. periods without excitation, whereas the red dots represent periods with excitation. There is a clear separation, with 19 micro-seconds' difference between the averages. This is the 'escapement effect'.

In contrast, **Figure 8** shows similar data for an excitation with 'half-down 1' equals 'half-down 2', at 0.5 mm. The period durations are now very close, with a difference of 4 micro-seconds between the averages: the 'escapement effect' is much reduced.

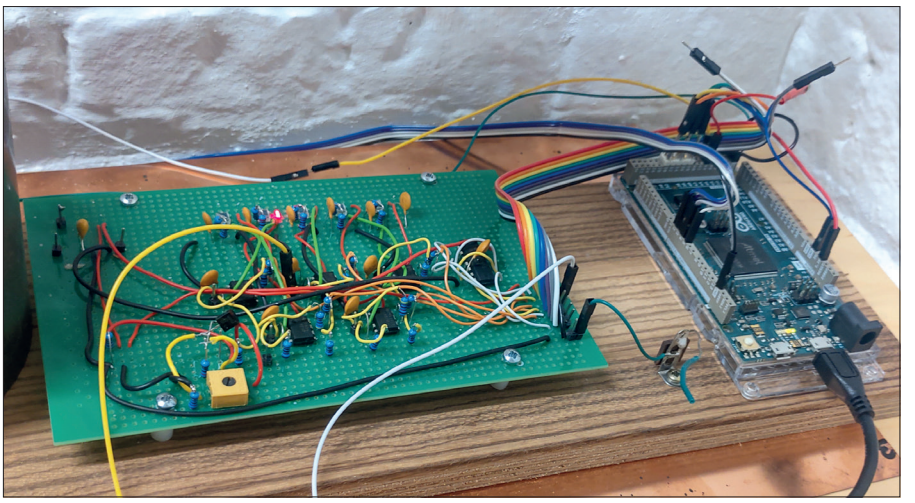

Figure 4. The detector board with an array of five photo-diodes, with the laser spot visible on the middle one.

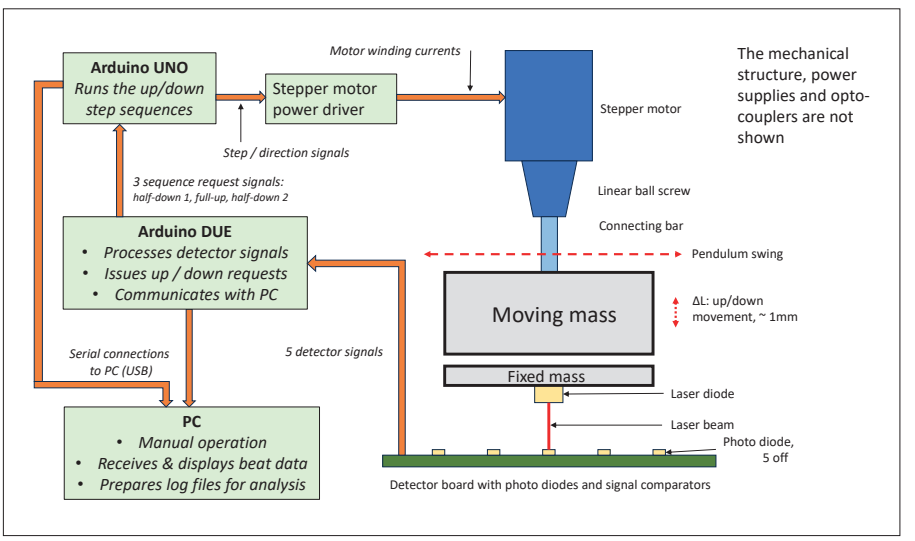

Figure 5. A schematic of the motion control system.

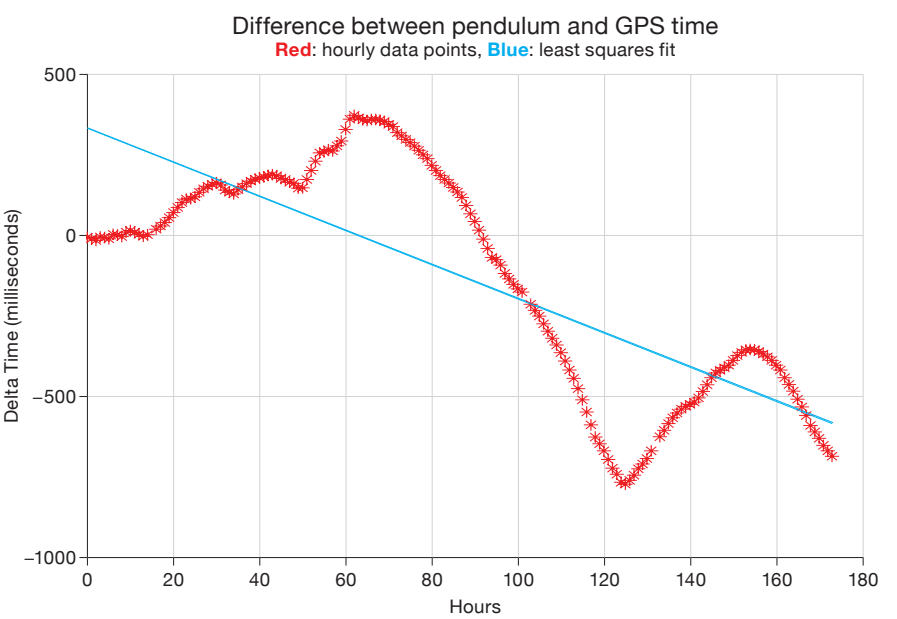

Figure 6. First results of a comparison of the pendulum with GPS time over a one-week period. The x-axis shows the elapsed time in hours, the y-axis shows the deviation between pendulum and GPS time in milliseconds. Data taken hourly.

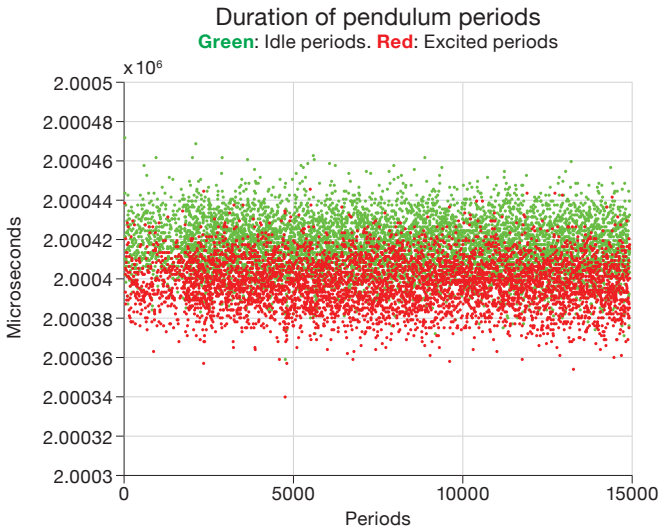

Figure 7. Illustration of the 'escapement effect' for 15,000 cycles. Green dots represent idle beats, red dots represent excited beats. We see an average separation of 19 micro-seconds.

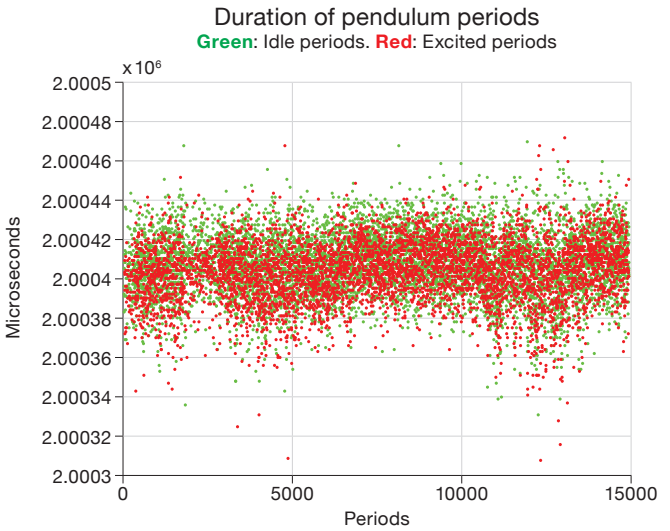

Figure 8. Reduction of the 'escapement effect' by tuning the excitation details. Green dots represent idle beats, red dots represent excited beats. The average separation is reduced to 4 micro-seconds.

The figures do not show some of the subtleties to do with start and end of excitation sequences. However, it is clear that, with some additional tuning, we can arrive at a situation where the excitation does not affect the period. The 'escapement effect' can be eliminated or, at least, made negligibly small.

Next Steps and Possible Directions

Our first objective will be to create an improved version of the pendulum. It should incorporate a number of technical and aesthetical improvements, and be built on a stand-alone base: something that can be displayed and turned into a clock. It should incorporate temperature compensation, which we will try to integrate in the length-variation mechanism.

In the longer term, one can think about more experimental work. It will be tempting to try to use the parametric excitation principle for a balance, for which the moment of inertia can be altered by varying the effective radius.

It is also interesting to speculate on the question of the ultimate accuracy that might be achieved. Obviously, the open-loop stepper-motor approach, while ideal for a first try, has its limits. For the next version, we may replace it with some form of eccentric mechanism. One can, however, think of far more accurate – and temperature-independent – means to control the position of the movable mass by the use of modern (but expensive) optical devices, as used in high-precision machining.

In Conclusion

I have described the construction of a parametrically-excited pendulum, which operates by variation of the effective pendulum length. The pendulum is experimental, made from easily available materials and components. First results on accuracy and stability, obtained from a week-long comparison with GPS-time, are encouraging.

Furthermore, this research has shown that, by judicious choice of the excitation strategy, it is possible to suppress the (equivalent of the) escapement effect. It is a good basis for further work and I hope that this article will give inspiration to others to explore this 'unusual pendulum', and thereby contribute to the general interest in horology.

Acknowledgements

Not having made clocks before, I was unsure if there would be an interest in the horological community for parametric excitation. I am therefore grateful to the British Horological Institute, in particular to Jim Arnfield FBHI, for the warm welcome given to my first, rather tentative email, and for the encouragement to write this article. Jim also gave me good advice that saved me time and effort, and commented on drafts of the article. Finally, I am indebted to Peter Wills for having shown that 'it can be done', and for drawing attention to the notion of the 'equivalent of the escapement effect'.

Appendix

In this appendix, following reference 3, we calculate – for an ideal model case – the variation of length required to sustain or grow the amplitude of the pendulum's motion.

In the following: m is the pendulum's mass; g is the acceleration due to gravity; l is the length; Δl is the length variation; v is the linear velocity at the bottom point; φ is the angle of swing; E is the pendulum's energy; Q is the quality factor, and L is angular momentum.

At the bottom point, our mechanism raises the bob – instantaneously – by an amount Δl. In doing so, it acts against gravity and against centrifugal force. It performs a mechanical work equal to:

$$W_{Bottom} = \left(mg + \frac{mv^2}{l}\right)\Delta l \qquad (2)$$

At the extremum, the mechanism recovers work. Here, there is no centrifugal force, and for the work recovered from gravity we need to take the vertical component. Consequently, the work recovered equals:

$$W_{Extremum} = mg\cos(\varphi)\,\Delta l \qquad (3)$$

We now eliminate both v and φ by using the pendulum energy equation:

$$E = \frac{mv^2}{l} = mgl(1-\cos\varphi) \qquad (4)$$

This results in work performed:

$$W_{Bottom} = \left(mg + \frac{2E}{l}\right)\Delta l \tag{5}$$

And work recovered:

$$W_{Extremum} = \left(mg + \frac{E}{l}\right)\Delta l \tag{6}$$

The net energy gain for one beat is then:

$$\Delta E = 3E\frac{\Delta l}{l} \tag{7}$$

The energy loss for one beat is expressed in terms of the quality factor Q as:

$$\Delta E_{Loss} = \frac{\pi E}{Q} \tag{8}$$

The condition for the pendulum to sustain or increase its amplitude is that ΔE is equal to or greater than ΔE_{Loss}. This condition is fulfilled for:

$$\frac{\Delta l}{l} \geq \frac{\pi}{3Q} \tag{9}$$

NOTE: The gain in kinetic energy at the bottom point can also be described as an effect of conservation of angular momentum $L=m\ l\ v$. The pendulum is a rotating system, so that L is conserved over the event of raising the bob.

This implies that $l_{before}\ v_{before} = l_{after}\ v_{after}$. In the same way a pirouette skater increases speeds by pulling in arms, the pendulum increases its velocity by becoming shorter.

We can rewrite the equation as:

$$\Delta v = \frac{\Delta l}{l} v \tag{10}$$

The energy gain then is:

$$\Delta E = \Delta\left(\frac{mv^2}{2}\right) = mv\Delta v = \frac{mv^2}{l}\Delta l \tag{11}$$

Which is the same result obtained above, in the expression for W_{Bottom}, second term.

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